Whiz Kids²

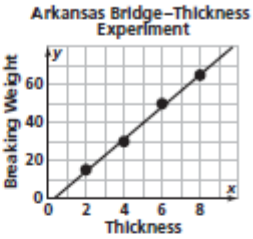
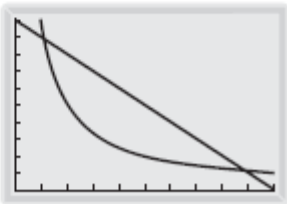
8th Grade Math Concepts

School districts have adopted the Connected Math curriculum and the concepts below are directly in line with that curriculum.

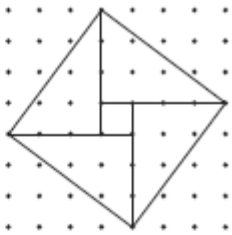
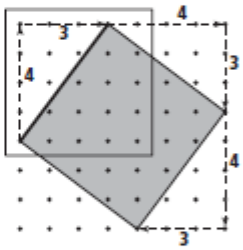
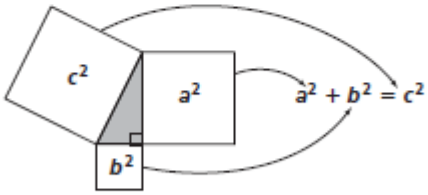
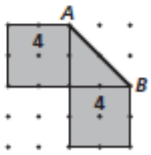
Another helpful resource is www.KhanAcademy.com where you can watch videos explaining each of the following concepts.

Contents

Linear and Inverse Variation

Important Concepts	Examples
Mathematical Models An equation or a graph that describes, at least approximately, the relationship between two variables is a mathematical model. A mathematical model may allow you to make reasonable guesses for values between and beyond the known data points.	**Modeling Bridge Thickness and Strength** **1.** Collect data by simulating how much weight a bridge can hold with various layers of thickness. **2.** Plot the data and draw a line to model the pattern of the data. **3.** Find an equation to model the data. For example, $y = 8x$ (since 0 thickness should suggest 0 strength). **4.** Use the equation $y = 8x$ to predict the breaking weights for other bridges: $\frac{\text{thickness in layers}}{\text{strength in penny load}}$ $\frac{3.5}{28}$ $\frac{7}{56}$ $\frac{10}{80}$ 
Linear Relationships In previous units, students learned how to recognize, represent symbolically, and analyze linear relationships. Many questions about linear relationships can be answered by solving equations of the form $c = mx + b$. The problems in this unit are designed to promote review and extension of these skills.	The **rate of change** of y in the equation $y = mx + b$ is the **slope** of its graph. In particular, m, the **coefficient** of x, indicates that constant ratio: $\frac{\text{change in } y}{\text{change in } x}$. The constant term b indicates the ***y*-intercept** $(0, b)$ of the graph. $5x - 3 = 7x - 2$ may be solved • by graphing (or making tables for) $y = 5x - 3$ and $y = 7x - 2$ and looking for a common solution. • by using Properties of Equality. $5x - 3 = 7x - 2$ $-3 = 2x - 2$ (subtract $5x$ from each side) $-1 = 2x$ (add 2 to each side) $-\frac{1}{2} = x$ (divide each side by 2)
Direct Variation Direct variation models are those that can be expressed with equations in the form $y = kx$. In a table of data, students may notice $\frac{y}{x} = k$. This is the same relationship as $y = kx$.	This is a special case of linear relationship in which the y-intercept is equal to zero.
Inverse Variation Inverse variation models are those that can be expressed with equations in the form $y = \frac{k}{x}$. It is important to realize that inverse variation gives a non-linear pattern of change. In a table of data, students may notice the pattern $xy = k$, where k is a constant. This is the same relationship as $y = \frac{k}{x}$.	Dividing by an increasing variable has a different effect than does subtracting an increasing variable. This fact is revealed by contrasting graphs of $y = 10 - x$ (the line) and $y = \frac{10}{x}$ (the curve). Notice that there is no solution for y when $x = 0$ in $y = \frac{10}{x}$. 

Pythagorean Theorem

Important Concepts	Examples
Finding Area Students find areas of squares drawn on grids. One method is to subdivide the square and add the areas of the component shapes. Another method is to enclose the square in a rectangle and subtract the area outside the figure from the area of the rectangle.	Area of tilted square = Area of 4 triangles + 1 small square $= 4\,[\frac{1}{2}(3 \times 4)] + 1$ = 25 square units 
Square Roots If the area of a square is known, its side length is the number whose square is the area. Some of these lengths are not whole numbers, so we use the $\sqrt{\ }$ symbol.	The area of the tilted square is 10 square units, so the side of the tilted square is $\sqrt{10}$ units. 
Estimating Square Roots Students develop benchmarks for estimating square roots.	$\sqrt{5}$ is between 2 and 3 because $2^2 < 5 < 3^2$. It is closer to 2. Try $2.25 \div 2.25^2 = 5.06$. So $\sqrt{5}$ is between 2 and 2.25, but closer to 2.25. Try 2.24 ($2.24^2 = 5.0176$), even closer. Continue until the desired accuracy is obtained.
Finding Distances To find various lengths of line segments, students begin by drawing a square that is associated with the length.	The line segment shown is the side of a square with an area of 25 square units, so the segment has length $\sqrt{25}$, or 5. 
Pythagorean Theorem In a right triangle, the sum of the squares of the lengths of the two legs is equal to the square of the length of the longest side, called the hypotenuse. Symbolically, this is $a^2 + b^2 = c^2$, where a and b are the lengths of the legs and c is the length of the hypotenuse.	
Length of Line Segment On a grid, the length of a horizontal or vertical line segment can be found by counting the distance. If a segment is not vertical or horizontal, it is possible to treat it as the hypotenuse of a right triangle. The Pythagorean Theorem is used to find the length of the hypotenuse.	The length of line segment AB can be the hypotenuse of a right triangle, c. $2^2 + 2^2 = c^2$, so $4 + 4 = 8 = c^2$. $\sqrt{8} = c$ 
Irrational Numbers A number that cannot be written as a fraction with an integer numerator or denominator is irrational. Decimal representations of irrational numbers never end and never show a repeating pattern for a fixed number of digits.	The numbers $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, and π are examples of irrational numbers. $\sqrt{2}$ is 1.41421356237.... The decimal part goes forever without any pattern of fixed length that repeats.

Exponential Relationships

Important Concepts	Examples

Exponential Growth

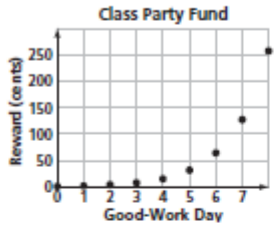
An exponential pattern of change involves patterns that are based on multiplication and can often be recognized in a verbal description of a situation or in the pattern of change in a table of (x, y) values.

The increasing rate of growth is reflected in the upward curve of the plotted points.

Suppose a reward is offered. At the start, 1¢ is put in a fund. On the first day, 2¢ is added; on the second day, 4¢ is added; and on each succeeding day, the reward is doubled. How much money is added on the eighth day?

Class Party Fund

Good-Work Day	Reward (cents)
0 (start)	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	?



Growth Factor

A constant factor can be obtained by dividing each successive y-value by the previous y-value. This ratio is called the *growth factor* of the pattern.

In the example above, you multiply the previous award by 2 to get the new reward. This constant factor can also be obtained by dividing successive y-values: $\frac{2}{1} = 2$, $\frac{4}{2} = 2$, etc.

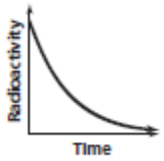
Exponential Equations

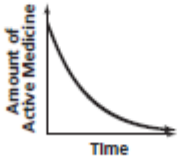
EXPONENTIAL GROWTH An exponential growth pattern, $y = a(b)^x$, increases slowly at first but grows at an increasing rate because its growth is multiplicative. The growth factor is b.

Day	Calculation	Reward (cents)
0	1	1
1	1×2, or 2^1	2
2	$1 \times 2 \times 2$, or 2^2	4
3	$1 \times 2 \times 2 \times 2$, or 2^3	8
⋮	⋮	⋮
6	$1 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$, or 2^6	256
⋮	⋮	⋮
n	$1 \times 2 \times 2 \times \cdots \times 2$, or 2^n	2^n

On the nth day, the reward, R, will be $R = 1 \times 2^n$. Because the independent variable in this pattern appears as an exponent, the growth pattern is called exponential. The growth factor is the *base*, 2. The *exponent*, n, tells the number of times the 2 is a factor.

EXPONENTIAL DECAY Exponential models describe patterns in which the value decreases. Decay factors result in decreasing relationships because they are less than 1.





$y = 50(\frac{1}{2})^n$

Rules of Exponents

The multiplicative structure of bases leads to:

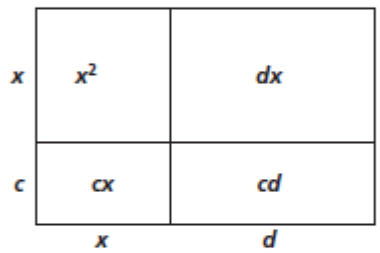
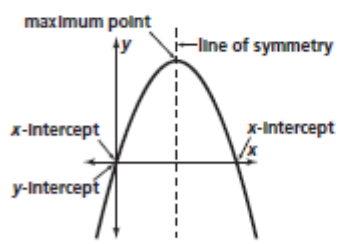
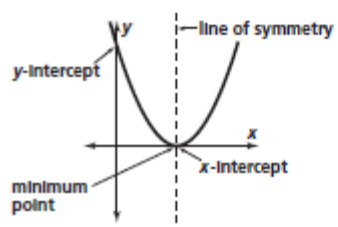
$(b^m)^n = b^{mn}$ — $(2^3)^2 = (2 \times 2 \times 2)^2 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) = 2^6$

$(b^m)(b^n) = b^{m+n}$ — $3^2 \times 3^3 = (3 \times 3) \times (3 \times 3 \times 3) = 3^5 = 243$

$(a^m b^m) = (ab)^m$ — $(2 \times 5)^2 = (2 \times 5) \times (2 \times 5) = (2 \times 2) \times (5 \times 5) = 2^2 \times 5^2$

$a^m/a^n = a^{m-n}$ — $5^3/5^2 = (5 \times 5 \times 5)/(5 \times 5) = 5^{3-2} = 5^1 = 5$

Quadratic Relationships

Important Concepts	Examples
Representing Quadratic Patterns of Change With Tables In linear relationships, the *first differences* of successive values are constant, indicating a constant rate of change. In quadratic relationships, first differences are not constant, but *second differences* are. The first difference is the rate at which y is changing with respect to x. The second difference indicates the rate at which *that rate* is changing. If the second differences are all the same, then the relationship is quadratic.	$y = 6(x - 2)^2$ $\begin{array}{cc} x & y \\ 0 & 24 \\ 1 & 6 \\ 2 & 0 \\ 3 & 6 \\ 4 & 24 \\ 5 & 54 \end{array}$ First Differences: $6 - 24 = {}^{-}18$; $0 - 6 = {}^{-}6$; $6 - 0 = 6$; $24 - 6 = 18$; $54 - 24 = 30$ Second Differences: ${}^{-}6 - ({}^{-}18) = 12$; $0 - ({}^{-}6) = 12$; $18 - 6 = 12$; $30 - 18 = 12$ The second differences are all 12, which indicates that the table represents a quadratic relationship.
Representing Quadratic Functions With Equations Traditionally, quadratic relationships are defined as relationships that have equations fitting the form $y = ax^2 + bx + c$, in which a, b, and c are constants, and $a \neq 0$. This form of the equation is called the *expanded form*. The emphasis is on observing that the equations contain an independent variable raised to the second power. It is also important to understand the *factored form* of such equations. Many quadratic equations can also be defined as functions whose y-value is equal to the product of two linear factors—the form $y = (ax + c)(bx + d)$, where $a \neq 0$ and $b \neq 0$. The power of this form is that it relates quadratic polynomials as products of linear factors.	The area of the rectangle below can be thought of as the product of two linear expressions, as the result of multiplying the width by the length, or as the sum of the area of the subparts of the rectangle.  $A = (x + c)(x + d)$ factored form $A = x^2 + cx + dx + d$ expanded form
Representing Quadratic Patterns of Change With Graphs The values in the equation affect the shape, orientation, and location of the quadratic graph, a parabolic curve. If the coefficient of the x^2 term is positive, the curve opens upward and has a minimum point. If negative, the curve opens downward and has a maximum point. The maximum or minimum point of a quadratic graph **(parabola)** is called the **vertex**. The vertex lies on the vertical *line of symmetry* that separates the parabola into halves that are mirror images. The vertex is located halfway between the ***x*-intercepts**, if the x-intercepts exist. The x-intercepts are mirror images of each other. The ***y*-intercept** is where the parabola crosses the y-axis.	 $y = -2x^2 + 8x$  $y = x^2 - 8x + 16$

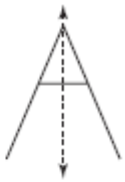
Symmetry and Transformations

Important Concepts and Examples

Types of Symmetry

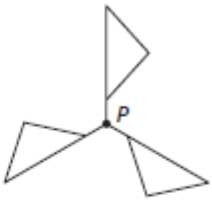
REFLECTION SYMMETRY A design has reflection symmetry if a reflection in a line makes an image that fits exactly onto the original figure or design.

This letter A has reflection symmetry because a reflection in the vertical line will match each point on the left half with a point on the right half. The vertical line is the line of symmetry.



ROTATION SYMMETRY A design has rotation symmetry if a rotation other than a full turn about a point makes an image that fits exactly onto the original figure or design.

This design has rotation symmetry because a rotation of 120° or 240° about point *P* will match each flag with another flag. Point *P* is the center of rotation. The angle of rotation is 120°, the smallest angle through which the design can be rotated to match with the original position.



TRANSLATION SYMMETRY A design has translation symmetry if a translation, or slide, maps the figure onto itself.

This figure is part of a translation-symmetric design. If this design continued in both directions, a slide to the right or left would match each flag with another.

Symmetry Transformations

Symmetry transformations—reflections, rotations, and translations—move points to image points so that the distance between any two original points is equal to the distance between their images. Students examine figures and their images under reflections, rotations, and translations, measuring key distances and angles. They use their findings to determine how they can specify a particular transformation so that another person could perform it exactly. They can also use this information about preserved distances to reason about shapes which have symmetry.

REFLECTIONS A reflection can be specified by giving the line of reflection.

The point *A* and its reflection image point *A'* lie on a line that is perpendicular to the line of symmetry and are equidistant from that line.

ROTATIONS A rotation can be specified by giving the center of rotation and the angle of the turn.

Point *B* and its image point *B'* are equidistant from the center of the rotation *P*. A point under a rotation "travels" on the arc of a circle, and the set of circles on which the points of the figure "travel" are concentric circles with *P* as their center. The angles formed by the vertex points of the figure and their rotation images all have measures equal to the angle of turn.

TRANSLATIONS A translation can be specified by giving the length and direction of the slide. Usually, an arrow with the appropriate length and direction is drawn.

If you draw the segments connecting a number of points to their images, the segments will be parallel and all the same length. The length is equal to the magnitude of the translation.

Congruent Figures

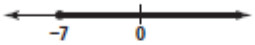
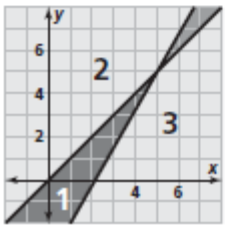
Figures of the same size and shape are congruent.

You can make an image of one figure which will fit exactly on top of the other by a combination of symmetry transformations.

Say It With Symbols

Important Concepts	Examples
Equivalent Expressions Students are deliberately presented with situations in which contextual clues can be interpreted in several ways to produce different, yet equivalent, equations.	*Find the number of 1-foot-square tiles, N, needed to make a border around a square pool with sides of length s feet.* Different conceptualizations of the situation can lead to different, yet equivalent, expressions for the number of tiles: $N = 4s + 4$ $N = 4(s + 1)$ $N = s + s + s + s + 4$ $N = 8 + 4(s - 1)$ $N = 2s + 2(s + 2)$ $N = (s + 2)^2 - s^2$. 
Revisiting the Distributive Property If an expression is written as a factor multiplied by a sum of two or more terms, the Distributive Property can be applied to *multiply* the factor by each term in the sum. If an expression is written as a sum of terms and the terms have a common factor, the Distributive Property can be applied to rewrite the expression as the common factor multiplied by a sum of two or more terms. This process is called *factoring*.	**multiply** $r(s + t) = rs + rt$ **factor** The Distributive Property allows students to group symbols (shown on the left side of the equation) or to expand an expression as needed (shown on the right side of the equation).
Checking for Equivalence Students may use contextual reasoning to decide if expressions are equivalent. Students may check whether equations have the same graphs and tables. Students should also be able to use the Distributive and Commutative Properties to show that expressions are equivalent.	By applying the Distributive Property $4(s + 1) = 4s + 4$. $8 + 4(s - 1)$ can be shown to be equivalent to $4s + 4$. $8 + 4(s - 1) = 8 + 4s - 4$ (Distributive Property) $= 8 - 4 + 4s$ (Commutative Property) $= 4 + 4s$ (Subtraction) $= 4s + 4$ (Commutative Property)
Solving Linear Equations Students have used tables or graphs to find solutions. They can solve simple linear equations using Properties of Equality. In this unit, students solve more complicated equations using Properties of Real Numbers.	$200 = 5x - (100 + 2x)$ $200 = 5x - (2x + 100)$ (Commutative Property) $200 = 5x - 2x - 100$ (Distributive Property) $200 = 3x - 100$ [Distributive Property, $5x - 2x = (5 - 2)x$] $300 = 3x$ (adding same to each side of an equation) $100 = x$ (dividing by the same on each side of an equation)
Solving Quadratic Equations Solving quadratic equations for x when $y = 0$ is equivalent to finding the x-intercepts on the graph. Students are also introduced to solving quadratic equations by factoring. The connection is made between the linear factors of a quadratic expression and the x-intercepts of the graph of a quadratic equation.	If $y = 2x^2 + 8x$, then the values of x when $y = 0$ can be obtained by rewriting the equation in the equivalent form of $2x(x + 4) = 0$. This product can be zero only if one of the factors is equal to 0. Solve $2x = 0$ and $x + 4 = 0$. Thus, $x = 0$ or $x = -4$. The x-intercepts are (0, 0) and (−4, 0). If $0 = x^2 + 5x + 6$, we write $x^2 + 5x + 6$ in factored form $(x + 2)(x + 3)$ and then solve $0 = (x + 2)(x + 3)$. Thus $x + 2 = 0$ or $x = -2$, and $x + 3 = 0$ or $x = -3$. The solutions of $x^2 + 5x + 6 = 0$ are $x = -3$ and $x = -2$.

The Shapes of Algebra

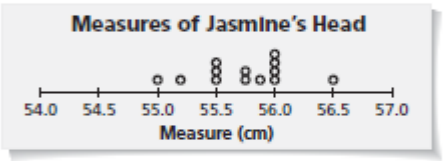
Important Concepts	Examples
Linear Inequalities A relation of inequality between two quantities, in which each quantity is a linear expressions, is called a linear inequality.	$3x + 22 < 8x + 7$ or $3x + 4y < 12$
Solving Linear Inequalities Solving an inequality is much like solving linear equations. The rules for operations with inequalities are identical to those for equations, with one exception. When multiplying (or dividing) an inequality by a negative number, the direction of the inequality is reversed.	$5x + 7 \leq 42$ $5x \leq 35$ $x \leq 7$ Solving this inequality is similar to solving $5x + 7 = 42$. The operations $(+, -, \times, \div)$ are applied to both sides. We usually show this solution on a number line.  $-5x + 7 \leq 42$ $-5x \leq 35$ $x \geq -7$ Reverse in the direction of the inequality sign. 
Solving Systems of Linear Equations Solving a system means finding all solutions that satisfy all equations in the system. There are a variety of techniques available for solving systems of two linear equations in two unknowns.	
GRAPHIC SOLUTION OF SYSTEMS This method involves producing straight-line graphs for each equation and then reading coordinates of intersection points as the solution(s).	**EQUIVALENT FORM** The equations in a system can each be changed to $y = ax + b$ form. For $\begin{cases} y = -2x + 5 \\ y = 3x - 5 \end{cases}$ set the two expressions for y equal to each other. This eliminates a variable and gives $(-2x + 5) = (3x - 5)$. So $5x = 10$, or $x = 2$. Find the corresponding y-value by substituting. $y = -2(2) + 5 = 1$ The solution is (2, 1).
SOLVING SYSTEMS BY SUBSTITUTION In the system $\begin{cases} 3x + 5y = 8 \\ 6x + y = 7 \end{cases}$, the second equation can be rewritten as $y = 7 - 6x$. Use this information about y and the first equation, $3x + 5(7 - 6x) = 8$. Now solve this equation with one unknown with methods from earlier work to reveal $x = 1$ and then $y = 7 - 6(1)$ or $y = 1$.	**SOLVING SYSTEMS BY LINEAR COMBINATION** Another method relies on two basic principles: **1.** Multiplying a linear equation by the same (non-zero) number does not change the set of solutions. **2.** The solution is unchanged if one of the equations is replaced by a new equation formed by adding the two original equations. For example: $\begin{cases} 3x + 5y = 8 \\ 6x + y = 7 \end{cases}$ is equivalent to $\begin{cases} -6x - 10y = -16 \\ 6x + y = 7 \end{cases}$ $-9y = -9$, by adding the two equations. You can see that $y = 1$ and that $x = 1$.
Solving Systems of Linear Inequalities Systems of inequalities tend to have infinite solution sets as well. The solution to a system of distinct, non-disjoint linear *inequalities* is the intersection of two half-planes, which contain infinitely many points.	In general, there are four regions suggested by a system of linear inequalities such as $\begin{cases} y < y \\ y > x - 5. \end{cases}$  Region 1 contains the solutions to the system. Points in Regions 2 and 3 satisfy one, but not both of the inequalities. Region 4 satisfies neither inequality.

Samples and Populations: Statistical Evaluation

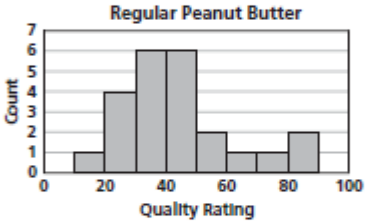
Important Concepts	Examples
The Process of Statistical Investigation This process involves four parts: posing a question, collecting the data, analyzing the distribution, and interpreting the analysis in light of the question. When completed, students need to communicate the results.	Students need to think about the process of statistical investigation whether they are collecting their own data or using data provided for them.
Distinguishing Different Types of Data An *attribute* is a name for a particular characteristic of a person, place, or thing about which data are being collected. There are two general kinds of data values: categorical and numerical.	We can have the attribute of *kind of peanut butter* to characterize whether a peanut butter is natural or regular, or the attribute of *quality rating* to characterize the quality (using a number) of a given type of peanut butter. Categorical values are "regular" or "natural" for the kind of peanut butter. Numerical values are the numbers used in the quality ratings for peanut butter.

Making Sense of a Data Set Using Different Representations

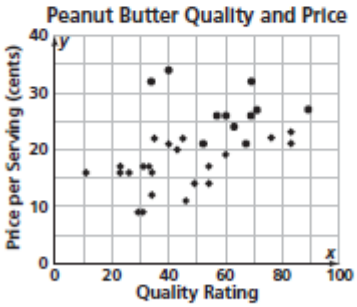
DOT PLOT (OR LINE PLOT) Each case is represented as a dot (or an "x") positioned over a labeled number line.



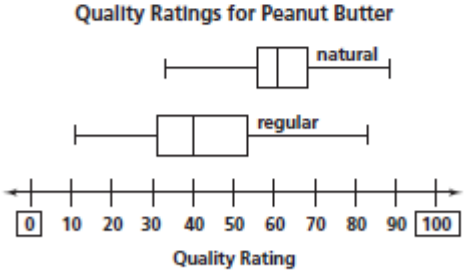
HISTOGRAM The size of the bar over that interval shows the frequency data values in each interval along the range of data values; frequencies may be displayed as counts or percentages.



SCATTERPLOT The relationship between two different attributes is explored by plotting values of two numeric attributes on a Cartesian coordinate system.



BOX-AND-WHISKER PLOT The box plot is divided into quartiles and displays the properties of distribution, such as symmetry or skewness. This plot was developed largely because comparing data using frequency bar graphs can often be confusing, especially if one is comparing more than two bar graphs.



Exploring the Concept of Sampling The essential idea behind sampling is to gain information about the whole by analyzing only a part of it. A primary issue in sampling is choosing a sample likely to be unbiased and predictive of the population.	To ensure fair samples, we try to choose random samples. Students consider other types of sampling strategies: convenience sampling, voluntary-response sampling, and systematic sampling. We want students to develop a sound, general sense about what makes a good sample size and how sample size affects the predictive quality of the sample.